Chapter 16+17 Review

1. E(mechanical) = 90 min   E(appearance) = 60 min
SD(mechanical) = 15 min   SD(appearance) = 5 min
var(mechanical) = 225 min   var(appearance) = 25 min

Cost(mechanical) = \$50 per/hr   cost(appearance) = \$6 per/hr

(a) The time to complete the two stages are independent

$$E(m+a) = E(m) + E(a) = 90 + 60 = 150 \text{ min}$$
$$var(m+a) = var(m) + var(a) = 225 + 25 = 250 \text{ min}$$
$$SD(m+a) = \sqrt{var(m+a)} = \sqrt{250} = 15.81 \text{ min}$$

$$\left.\begin{aligned} E(m+a) &= 150 \text{ min or } 2\tfrac{1}{2} \text{ hrs} \\ SD(m+a) &= 15.81 \text{ min or } .2635 \text{ hrs.} \end{aligned}\right\} \text{time}$$

converted 90 min to 1.5 hrs

(b) $E(am) = aE(m) = 50(1.5) = \$75$

converted 60 min to 1 hr.

$E(aA) = aE(A) = 6(1) = \$6$

$E(aM + aA) = E(aM) + E(aA) = 75 + 6 = \$81$

→

225 min converted to hrs

$$Var(aM) = a^2 var(M) = 50^2(3.75) = 9375$$

25 min converted to hrs

$$var(aA) = a^2 var(A) = 6^2\left(\frac{5}{12}\right) = 15$$

$$var(aM + aA) = var(aM) + var(aA) = 9375 + 15 = 9390$$

$$SD(aM + aA) = \sqrt{var(aM + aA)} = \sqrt{9390} = 96.90$$

So, $E(aM + aA) = \$81$
$SD(aM + aA) = \$96.90$ } Sum of Cost for both stages

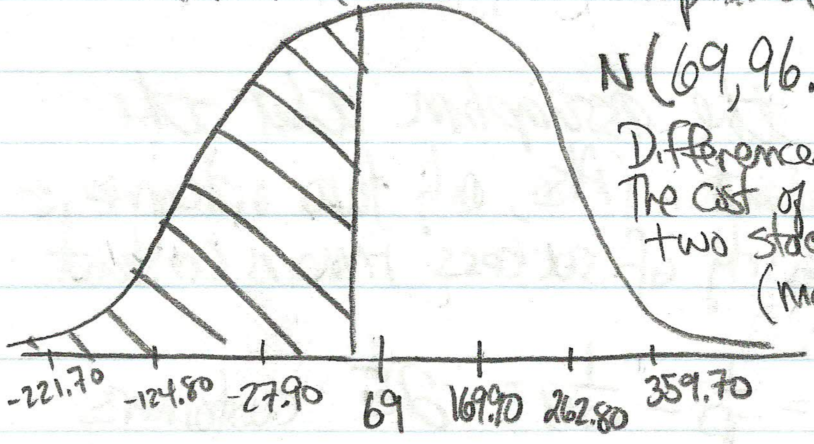
(c) $E(aM - aA) = E(aM) - E(aA) = 75 - 6 = \$69$

$$var(aM - aA) = var(aM) + var(aA) = 9375 + 15 = 9390$$

$$SD(aM - aA) = \sqrt{var(aM) + var(aA)} = \sqrt{9390} = \$96.90.$$

So, $E(aM - aA) = \$69$
$SD(aM - aA) = \$96.90$ } Difference in cost per between two stages.

(1) It is stated that both prep times (and therefore costs) are normally distributed. Also, it was already determined that both phases (stages) were independent.



$$Z = \frac{Y-\mu}{\sigma}$$

$$Z_0 = \frac{0-69}{96.90}$$

$$Z_0 = -.71$$

P-value = p(mechanical − appearance < 0)
$= p(Z < -.71)$
$= .238$

(2) a) $E(X) = 0(.05) + 1(.1) + 2(.2) + 3(.25) + 4(.3) + 5(.1)$
$= 2.95$ emails per day

b) $\mathrm{var}(X) = (0-2.95)^2(.05) + (1-2.95)^2(.1) + (2-2.95)^2(.2) + (3-2.95)^2(.25) + (4-2.95)^2(.3) + (5-2.95)^2(.1)$
$= 1.7475$

$SD(X) = \sqrt{\mathrm{var}(X)} = \sqrt{1.7475} = 1.32$ emails per day

(c) $E(aX) = aE(X) = 10(2.95) = 29.5$ min

(3) We will make the assumption that the trials are independent. (Also, only two outcomes, & that the probability of success remains constant with each trial).

(a) $\mu = \frac{1}{p} = \frac{1}{.04} = 25$ customers

(b) $P(X=x) = q^{x-1}p$

$P(X=7) = (.96)^6(.04)$

$= .0313$

(c) $P(X=x) = \binom{n}{x} p^x q^{n-x}$

${}_{10}C_3(.04)^3(.96)^7$

OR

binompdf(10,.04,3)

$P(X=3) = \binom{10}{3}(.04)^3(.96)^7$

$= .0058$

${}_{40}C_3(.04)^3(.96)^{37} + \ldots + {}_{40}C_{40}(.04)^{40}(.96)^0$

OR

$1-[p(X<3)]$

$\underbrace{1-[\text{binomcdf}(40,.04,2)]}_{1-.7855}$

(d) $P(X=x) = \binom{n}{x} p^x q^{n-x}$

$P(X=3,4,\ldots,40) = P(3) + p(4) + p(5) + \ldots + P(40)$

$= \binom{40}{3}(.04)^3(.96)^{37} + \binom{40}{4}(.04)^4(.96)^{36} + \ldots + \binom{40}{40}(.04)^{40}(.96)^0$

$= .2145$

e) $\mu = np = 275(.04) = 11$

$\sigma = \sqrt{npq} = \sqrt{275(.04)(.96)} = 3.25$

f) I would look to find a number of customers who buy the specialty clothes to be equal to or greater than the value at 2.5 standard deviations above the mean. Realize that before we can do this that you will need to check the Success/Failure Condition

$np \geq 10$ and $nq \geq 10$

$275(.04) \geq 10$ and $275(.96) \geq 10$

$11 \geq 10$ and $264 \geq 10$

Condition is satisfied therefore, we can use a Normal model

To then find the number of customers to put us at 2.5 SD above mean, solve the equation: $2.5 = \frac{y-11}{3.25} \Rightarrow y = 19.125$,

so if 19 or more customers buy the specialty clothes, that would appear to make the 4% value too low.

4. The data represent a Bernoulli Trial. Also, we can use a Normal Model:

Success/Failure Condition

$np \geq 10$ and $nq \geq 10$

$500(.5) \geq 10$ $\quad$ $500(.5) \geq 10$

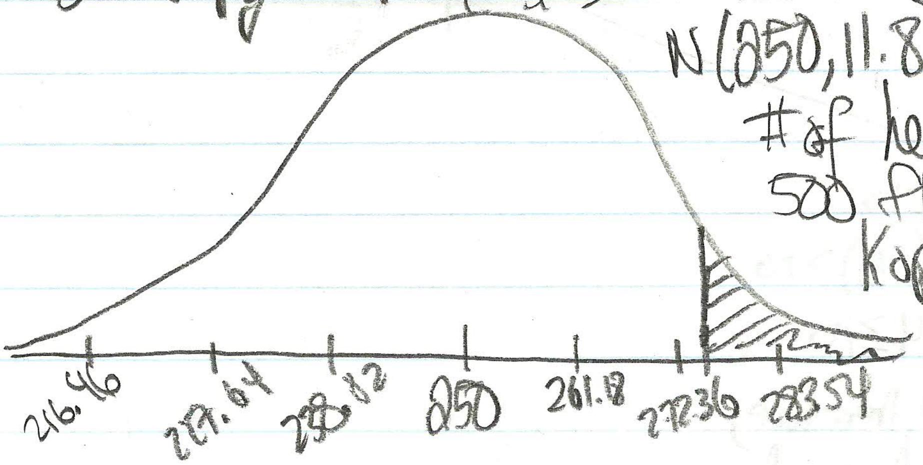
$250 \geq 10$ and $250 \geq 10$ ← Both values are greater than 10, we can use the normal curve to approximate

$\mu = np = 500(.5) = 250$

$\sigma = \sqrt{npq} = \sqrt{500(.5)(.5)} = 11.18$

N(250, 11.8)

# of heads in 500 flips of Kopec



$z = \frac{y-\mu}{\sigma}$

$z = \frac{280-250}{11.18}$

$z = 2.68$

$$\begin{aligned} \text{p-value} &= P(y > 280) \\ &= P(z > 2.8) \\ &= .0026 \end{aligned}$$

Very unusual results, seems to indicate a biased coin.

(5)

$E(A+C+B) = E(A) + E(C) + E(B) = 36 + 22 + 8 = 66$

$var(A+C+B) = Var(A) + var(C) + Var(B) =$

$(2.5)^2 + (1.8)^2 + (.75)^2 =$

$6.25 + 3.24 + .5625 =$

10.0525

$SD(A+C+B) = \sqrt{var(A+B+C)} = \sqrt{10.0525}$

$= 3.17$